

Semiparametric Approach to Hammerstein System Identification

W. Greblicki and G. Mzyk

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Abstract: Two nonparametric algorithms to recover a nonlinear characteristic in a Hammerstein system are proposed. Both are obtained by incorporating a parametric component into the kernel nonparametric algorithm. For small number of observations, their identification errors are smaller than that of the purely nonparametric algorithm.

1. INTRODUCTION

A problem of nonlinearity recovering in dynamic system with composite structure is extensively studied in the literature. Nonparametric identification algorithms are a proper tool if the *a priori* information about the system is small, less than parametric. It is however known that, due to small *a priori* information, nonparametric algorithms have rather large error for small number of observations. We show how this drawback can be eliminated or at least significantly reduced. We do it by incorporating a parametric component into a purely nonparametric algorithm. The resulting estimate has, thus, two terms. A parametric term is active for small number of observations while a nonparametric one dominates when the number is large.

2. THE IDENTIFICATION PROBLEM

A discrete-time Hammerstein system with input U_i and output Y_i , see Fig. 1, consists of a memoryless nonlinear subsystem followed by a linear dynamic one. The first part has a characteristic m which means that $V_i = m(U_i)$ while the second one is described by the following state equation:

$$\begin{cases} X_{i+1} = AX_i + bV_i \\ W_i = c^T X_i, \end{cases}$$

where X_i is the state vector. Its output signal is disturbed by Z_i due to which $Y_i = W_i + Z_i$ is output of the whole system.

We assume that $\{\dots, U_{-1}, U_0, U_1, \dots\}$ is a stationary random process such that

$$EU_i^2 < \infty. \quad (1)$$

The probability density of U_n 's denoted by f is completely unknown. Since the characteristic m satisfies the following restriction:

$$|m(u)| \leq \alpha + \beta|u|, \quad (2)$$

where $\alpha, \beta > 0$, are some unknown finite constants, we have $EV_i^2 < \infty$. Owing to this and the fact that, by assumption, the linear subsystem is stable, W_n 's are random variables and $\{\dots, W_{-1}, W_0, W_1, \dots\}$ is a stationary random process. We assume that the disturbance $\{\dots, Z_{-1}, Z_0, Z_1, \dots\}$ is also a stationary random process independent of the input signal and such that $EZ_i = 0$.

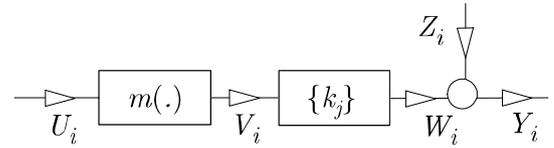


Fig. 1. The Hammerstein system.

Both the nonlinear characteristic m and the impulse response $\{k_j\}$ of the linear part are unknown and our goal is to recover them from input-output observations $(U_0, Y_0), (U_1, Y_1), \dots, (U_n, Y_n)$ of the whole system. Since both U_i and Y_i are random variables, U_i by a straight assumption while Y_i due to (1), (2) and stability of the dynamic subsystem, our identification problem can be posed in probabilistic terms.

The class of all characteristics satisfying (2) is so wide that cannot be parameterized. Neither can be the family of all possible stable subsystems, since its order, i.e., the dimension of the state vector X_i , is unknown. Thus, the *a priori* information, and consequently our identification problem, about both subsystems is nonparametric.

Our additional restriction that $\text{var } Z_i < \infty$ is rather standard. Not to confine generality of our further considerations but to simplify them, we assume, moreover, that

$$EU_i = 0, \quad (3)$$

$$f \text{ is an even function.} \quad (4)$$

$$m \text{ is an odd function,} \quad (5)$$

We need (3), (4), and (5) only to assure that $EV_i = 0$, and consequently $EX_i = 0$.

3. NONPARAMETRIC KERNEL ALGORITHM

The motivation for nonparametric identification algorithms is the following fact:

$$E\{Y_{i+1}|U_i = u\} = c^T AEX_i + c^T bm(u) = \mu(u),$$

where $\mu(u) = \alpha m(u)$, and where $\alpha = c^T b$ is unknown. Therefore to recover $\mu(u)$, i.e. $m(u)$ up to some unknown constant α , it suffices to estimate the regression function $E\{Y_{i+1}|U_i = u\}$ in a nonparametric way. The kernel nonparametric algorithm has the following form (see Nadaraya [1964] and Watson [1964]):

$$\hat{\mu}(u) = \frac{\sum_{i=1}^n Y_{i+1} K\left(\frac{u - U_i}{h_n}\right)}{\sum_{i=1}^n K\left(\frac{u - U_i}{h_n}\right)}. \quad (6)$$

The kernel K is such, that

$$\begin{aligned} \sup_u |K(u)| &< \infty, \\ \int |K(u)| du &< \infty, \\ uK(u) &\rightarrow 0 \text{ as } |u| \rightarrow \infty \end{aligned}$$

while the number sequence $\{h_1, h_2, \dots\}$ is selected to satisfy the following restrictions:

$$\begin{aligned} h_n &\rightarrow 0 \text{ as } n \rightarrow \infty, \\ nh_n &\rightarrow \infty \text{ as } n \rightarrow \infty. \end{aligned}$$

It is known that the estimate is consistent, i.e., that

$$\hat{\mu}(u) \rightarrow \mu(u) \text{ as } n \rightarrow \infty \text{ in probability} \quad (7)$$

at every point u at which both m and f are continuous and, moreover, $f(u) > 0$, see Greblicki and Pawlak [1986], [1987], and also [1989], [1994], Krzyżak [1990], Krzyżak and Partyka [1993]. Moreover, if both m and f are differentiable at a point u and $h_n \sim n^{-1/2}$,

$$|\hat{\mu}(u) - \mu(u)| = O(n^{-1/4}) \text{ in probability.} \quad (8)$$

Numerical simulation unveils, however, some unwanted property of the estimate. For small number of observations, the Mean Integrated Squared Error

$$\text{MISE} = \int E(\hat{\mu}(u) - \mu(u))^2 du$$

is large and diminishes rather slowly. To overcome this disadvantage, we modify (6) and present two new algorithms. For small n , both behave much better than (6), for large n are close to (6).

For small n the algorithms pay main attention to the linear part of in m , for large n they are targeted at the nonlinearity. By the linear part we mean the best linear approximation ρu of $\mu(u)$, i.e., ρu with ρ minimizing $E(\mu(U_0) - \rho U_0)^2$, or, equivalently, $E(Y_1 - \rho U_0)^2$. The optimal approximation, i.e., the linear part of the nonlinearity is certainly $\rho^* u$ with $\rho^* = E\{Y_1 U_0\} / EU_0^2$.

Each our algorithm has two components, one parametric and one nonparametric. The parametric one estimates the linear part while the other recovers the nonlinearity. The parametric one decreases MISE for small number of observations while the nonparametric component assures consistency.

Both algorithms are still nonparametric in nature. Nevertheless they are called semiparametric like all various nonparametric estimates with incorporated parametric components studied in the statistical literature, see e.g., Härdle [1990]. In general, the parametric component does not change asymptotic properties, but is expected to improve the behavior of the estimate for small and moderate number of observations.

4. SEMIPARAMETRIC ALGORITHMS

Estimating ρ^* in the following way:

$$\hat{\rho} = \frac{\sum_{i=1}^n Y_{i+1} U_i}{\sum_{i=1}^n U_i^2},$$

we define

$$\hat{p}(u) = \hat{\rho}u \quad (9)$$

as an estimate of $\rho^* u$. The remaining part of $\mu(u)$, i.e., $q(u) = \mu(u) - \rho^* u$, we estimate with the kernel estimate, see (6), in the following way:

$$\hat{q}(u) = \frac{\sum_{i=1}^n (Y_{i+1} - \hat{\rho}U_i) K\left(\frac{u - U_i}{h_n}\right)}{\sum_{i=1}^n K\left(\frac{u - U_i}{h_n}\right)}.$$

Therefore, our first estimate has the following form:

$$\hat{\mu}_1(u) = \hat{p}(u) + \hat{q}(u). \quad (10)$$

Its first component recovers $\rho^* u$ in a parametric way, the other estimates the remaining part $q(u) = \mu(u) - \rho^* u$ in a nonparametric way. Advantages of the first dominate for small n , of the other for large.

From (7), it follows that $\hat{q}(u) \rightarrow q(u)$ as $n \rightarrow \infty$ in probability at every point u at which both m and f are continuous and, moreover, $f(u) > 0$. Since $\hat{r}(u) \rightarrow \rho^* u$ as $n \rightarrow \infty$ in probability at every point u , our semiparametric estimate converges to the nonlinear characteristic, i.e.,

$$\hat{\mu}_1(u) \rightarrow \mu(u) \text{ as } n \rightarrow \infty \text{ in probability}$$

at every point u at which both m and f are continuous and $f(u) > 0$.

Our second algorithm is of the following form:

$$\hat{\mu}_2(u) = \lambda_n \hat{p}(u) + (1 - \lambda_n) \hat{\mu}(u), \quad (11)$$

with $\lambda_n \rightarrow 0$ as $n \rightarrow \infty$. It is just a linear combination of $\hat{p}(u)$ recovering the linear part and $\hat{\mu}(u)$ estimating $\mu(u)$. It is obvious that consistency of $\hat{\mu}(u)$ implies consistency of $\hat{\mu}_2(u)$, which means that

$$\hat{\mu}_2(u) \rightarrow \mu(u) \text{ as } n \rightarrow \infty \text{ in probability}$$

at every point u at which both m and f are continuous and $f(u) > 0$. If the balance in the combination is proper, for small n , MISE should decrease at the rate typical for parametric inference.

It is clear that the parametric component $\hat{p}(u)$ in (10) converges to $\rho^* u$ in probability at the rate $n^{-1/2}$ while the nonparametric one $\hat{q}(u)$ to $q(u)$ at the rate typical for nonparametric inference being slower than $n^{-1/2}$, see, e.g., (8). The same is with (11), $\hat{r}(u)$ converges to $\rho^* u$ faster than $\hat{\mu}(u)$ to $\mu(u)$. Therefore for small n , where parametric components dominate nonparametric and errors of both (10) and (11) are expected to be smaller than that of (6).

5. IDENTIFICATION OF THE LINEAR DYNAMIC SUBSYSTEM

In general, the linear dynamic component with the impulse response $\{k_j\}_{j=1}^\infty$, i.e.,

$$W_n = \sum_{j=1}^\infty k_j V_{n-j}, \quad k_1 = 1, \quad (12)$$

cannot be described by the model, which includes a finite number of parameters. Therefore, almost every purely

parametric method is characterized by a systematic approximation error. We will show that this error can be eliminated with the help of additional nonparametric term in the estimate. Let us consider the class of 2nd-order models with the transmittances

$$K(z; a_1, a_2) = \frac{z}{z^2 - a_1 z - a_2}, \quad (13)$$

and $\mathbf{a}^* = (a_1^*, a_2^*)^T$, where a_1^* and a_2^* are best approximation of the true dynamic object (12) in the class (13), i.e.,

$$a_1^*, a_2^* = \arg \min_{a_1, a_2} E(W_n - a_1 W_{n-1} - a_2 W_{n-2})^2$$

and

$$\{k_j^*\} = \mathcal{Z}^{-1}\{K(z; a_1^*, a_2^*)\}$$

be the impulse response of this model. Traditionally, parameters a_1^* , and a_2^* are identified by the instrumental variables (*iv*) method (see Wong and Polak [1967] and Stoica and Söderström [2002]):

$$\hat{\mathbf{a}}^{(iv)} = (\hat{a}_1^{(iv)}, \hat{a}_2^{(iv)})^T = (\Psi_n^T \Phi_n)^{-1} \Psi_n^T \mathbf{Y}_n, \quad (14)$$

where $\Phi_n = (\phi_1, \phi_2, \dots, \phi_n)^T$, $\phi_i = (Y_{i-1}, Y_{i-2})^T$, $\mathbf{Y}_n = (Y_1, Y_2, \dots, Y_n)^T$ and $\Psi_n = (\psi_1, \psi_2, \dots, \psi_n)^T$ is the matrix of appropriately selected instruments, such that the following two conditions are fulfilled:

- (C1) $\text{Plim } \Psi_n^T \Phi_n$ exists and is not singular,
- (C2) $\text{Plim } \Psi_n^T \tilde{\mathbf{Z}}_n = 0$, where $\tilde{\mathbf{Z}}_n = (\tilde{Z}_1, \tilde{Z}_2, \dots, \tilde{Z}_n)^T$ and $\tilde{Z}_i = Z_i - a_1 Z_{i-1} - a_2 Z_{i-2}$.

The impulse response can be estimated indirectly as

$$\{\hat{k}_j^{(iv)}\} = \mathcal{Z}^{-1}\{K(z; \hat{a}_1^{(iv)}, \hat{a}_2^{(iv)})\}. \quad (15)$$

Since it holds that (see Wong and Polak [1967] and Hasiewicz and Mzyk [2009])

$$\hat{\mathbf{a}}^{(iv)} \xrightarrow{n} \mathbf{a}^* \text{ in probability,} \quad (16)$$

under continuity of the transform \mathcal{Z}^{-1} , we get also

$$\hat{k}_j^{(iv)} \xrightarrow{n} k_j^* \text{ in probability,} \quad (17)$$

$j = 1, 2, \dots$ Let us, however, emphasize that $k_j^* \neq k_j$ whenever the parametric model is not correct. For n large, the true elements $\{k_j\}$ of the impulse response of the linear dynamics can be recovered successfully by the nonparametric way, using input-output cross-correlation analysis (Greblicki and Pawlak [1986]). The nonparametric estimate has the form

$$\hat{k}_j^{(corr)} = \frac{\hat{\varkappa}_j}{\hat{\varkappa}_1}, \text{ where } \hat{\varkappa}_j = \frac{1}{n-1} \sum_{i=1}^{n-j} Y_{i+j} U_i. \quad (18)$$

Although the estimate (18) is consistent, i.e.,

$$\hat{k}_j^{(corr)} \xrightarrow{n} k_j \text{ in probability,}$$

it has huge variance-error if $n - j$ is small. Therefore, analogously to (11) we propose to combine (15) with (18) as follows

$$\hat{k}_j^{(semi)} = \lambda_n \hat{k}_j^{(iv)} + (1 - \lambda_n) \hat{k}_j^{(corr)} \quad (19)$$

with $\lambda_n \rightarrow 0$ as $n \rightarrow \infty$.

6. TECHNICAL ASPECTS

In the algorithms presented in Sections 3 and 4 the nonlinear characteristic is identified completely independently of

the linear dynamic part. Similarly, in Section 5 the linear dynamics is identified independently of the static nonlinear block. Here we describe the ideas of the combined estimates (see Hasiewicz and Mzyk [2004]), in which the results of identification of one part of Hammerstein system support identification of the other.

6.1 Nonparametric instrumental variables

Most of results concerning strategies of generation of instrumental variables cannot be directly adopted from the linear system theory. The classical instruments (Wong and Polak [1967]) of the form

$$\psi_i^{(1)} = (\alpha V_{i-2}, \beta V_{i-3})^T \quad (20)$$

fulfill conditions (C1) and (C2) for each constants $\alpha, \beta \neq 0$, but they cannot be realized because the signal V_n is not accessible for measurement in Hammerstein system. On the other hand, the instruments based on the system input, e.g.,

$$\psi_i^{(2)} = (\alpha U_{i-2}, \beta U_{i-3})^T$$

generally does not assure fulfillment of (C1) and (C2). One of constructive solutions is

$$\psi_i^{(3)} = (\hat{V}_{i-2}, \hat{V}_{i-3})^T, \quad (22)$$

where $\hat{V}_i = \hat{\mu}(U_i)$ is nonparametric kernel estimate of V_i . The consistency of the estimate (14) with the instrumental variables as in (22) has been strictly proved in Hasiewicz and Mzyk [2009].

6.2 Kernel regression estimation with filtered output

In kernel method (6), the Hammerstein system is treated in fact as a nonlinear static element

$$Y_i = m(U_{i-1}) + \xi_i$$

with correlated output noise

$$\xi_i = \sum_{j=2}^{\infty} k_j m(U_{i-j}) + Z_i. \quad (23)$$

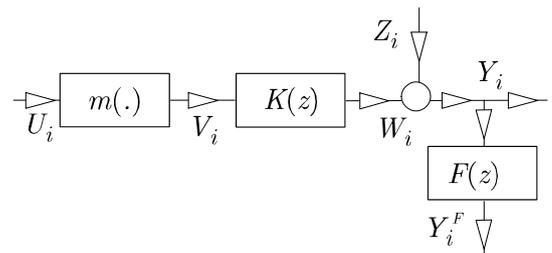


Fig. 2. The Hammerstein system with filtered output.

Since the component $\sum_{j=2}^{\infty} k_j m(U_{i-j})$ connected with dynamic part of the system usually dominates Z_i and has harmful influence on the kernel estimate for small n , let us introduce the generalized version of (6):

$$\hat{\mu}^F(u) = \frac{\sum_{i=1}^n Y_{i+1}^F K\left(\frac{u - U_i}{h_n}\right)}{\sum_{i=1}^n K\left(\frac{u - U_i}{h_n}\right)}, \quad (24)$$

where Y_i^F is filtered output (see Fig. 2) and $F(z)$ is additional linear filter. Resulting system, with the input

U_i and the output Y_i^F obviously belongs also to the class of Hammerstein systems, and has the same static characteristic m as the original one. It was shown in Mzyk [2007] and Mzyk [2009] that appropriate selection of the filter $F(z)$, e.g. $F(z) = K^{-1}(z)$ for invertible dynamics, allows to decrease the variance of (24) in comparison with (6). Since $K(z)$ is unknown, we recommend to apply the inverse of its AR(2) approximation, obtained in parametric way, i.e.,

$$Y_i^F = Y_i - \hat{a}_1^{(iv)} Y_{i-1} - \hat{a}_2^{(iv)} Y_{i-2}.$$

7. SIMULATION EXAMPLES

7.1 Recovering the nonlinear characteristic

In the simulation example, U_n is distributed uniformly on $[-1, 1]$, Z_n has a Gaussian distribution with zero mean and variance 0.25. The linear subsystem is described by the following equations: $X_{n+1} = aX_n + V_n$, $W_n = X_n$, with $a = 0.6$. Moreover, see Fig. 3,

$$m(u) = \begin{cases} u + 0.5 \sin \frac{10\pi u^2}{1+u^2} + 0.5, & \text{for } u \leq 0 \\ u + 0.5 \sin \frac{10\pi u^2}{1+u^2} - 0.5, & \text{otherwise.} \end{cases}$$

The kernel is a rectangular function equal to 1 for $|u| \leq 1$ and 0 otherwise. Moreover, in estimate (11),

$$\lambda_n = \frac{1}{1 + (n/200)^6}.$$

The quality of each examined algorithm is measured with MISE, e.g., for (6), $MISE = \int_{-1}^1 E(\hat{\mu}(u) - \mu(u))^2 du$.

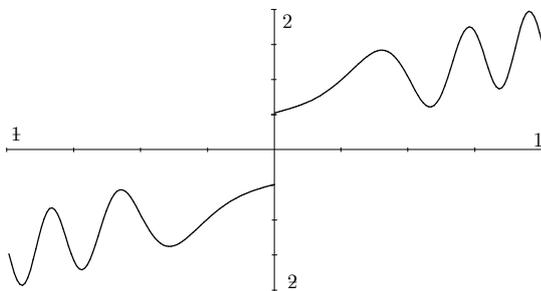


Fig. 3. The nonlinear characteristic m .

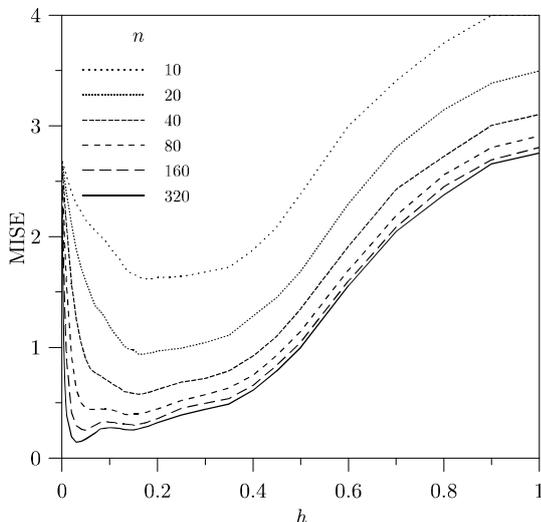


Fig. 4. MISE for nonparametric estimate (6).

It is well known that purely nonparametric estimate (6) is sensitive to improper selection of h_n . This phenomenon is shown in Fig. 4; h_n is replaced with h . It is obvious that too small h must be avoided. This disadvantageous effect is much weaker in semiparametric algorithms, see Figs. 5 and 6.

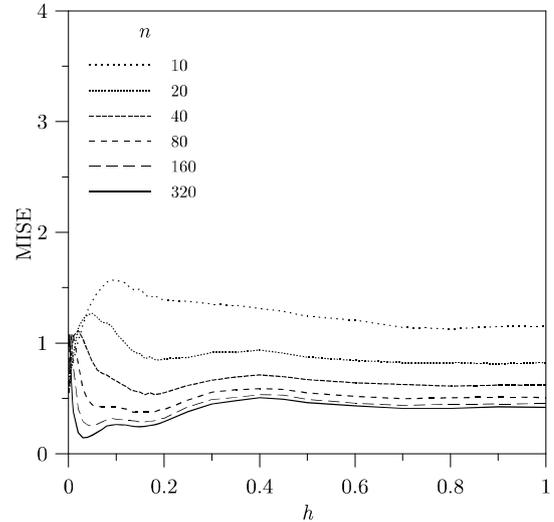


Fig. 5. MISE for semiparametric estimate (10).

To demonstrate what can be achieved by our algorithms, h_n has been selected optimally in nonparametric (6) as well as in semiparametric (10) and (11). Results are shown in Fig. 7. To get more comprehensive view, we have shown also the error for purely parametric algorithm (9). Superiority of (10) and (11) for n smaller than 80 is obvious.

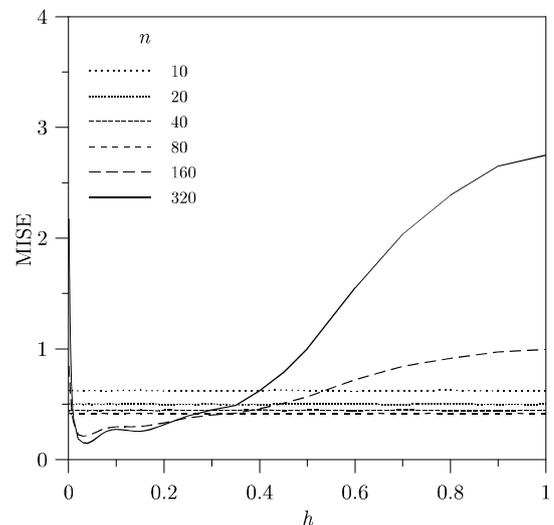


Fig. 6. MISE for semiparametric estimate (11).

In real situations selecting optimal h_n is clearly impossible. Therefore, for h_n selected rather arbitrarily as $0.8n^{-0.3}$, results are shown in Fig. 8. Semiparametric estimates are also superior.

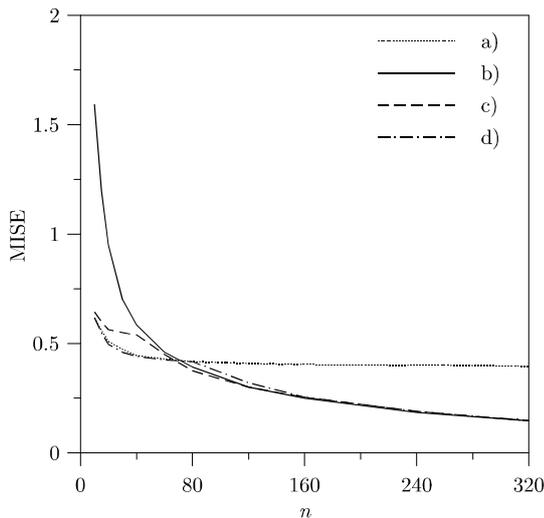


Fig. 7. MISE versus n ; h optimal; a) – (9), b) – (6), c) – (10), d) – (11).

7.2 Nonlinear recovering with data pre-filtering

For the system simulated in Section 7.1 we compared the estimates (6) and (24) for various n . We used the rectangular kernel and set $h_n = 0.8n^{-0.3}$ in both algorithms. The results are presented in Table 1. Thanks to data prefiltering the MISE has been reduced significantly.

Table 1. MISE for the kernel estimate with filtration and without.

N	$MISE(\hat{\mu}^F(u))$	$MISE(\hat{\mu}(u))$
100	0.08	0.54
300	0.05	0.32
500	0.03	0.16
1000	0.02	0.11
3000	0.01	0.06
5000	0.00	0.03

7.3 Estimation the impulse response

In the simulation presented in Section 7.1 we replaced the linear dynamic component with the FIR filter, including four nonzero elements of the impulse response, i.e., we set $k_1 = 1, k_2 = 0.75, k_3 = 0.5, k_4 = 0.25$, and $k_j = 0$ for $j > 4$. Since $\rho^* \neq 0$, one can show that

$$\text{cov}(U_i, V_i) \neq 0$$

and consequently the instruments generated according the rule

$$\psi_i = (U_{i-2}, U_{i-3})^T$$

fulfill conditions (C1) and (C2). The estimation error of both $k_j^{(iv)}$ and $k_j^{(corr)}$ has been computed as follows

$$\text{ERR}\{\hat{k}_j\} = \frac{\sum_{j=1}^{10} (\hat{k}_j - k_j)^2}{\sum_{j=1}^{10} k_j^2} \cdot 100[\%]$$

Obviously, a FIR object does not belong to the class of AR(2) models defined in (13), which allows to demonstrate the asymptotic approximation error of the parametric estimate and consistency of the nonparametric one (see Fig. 9). However, for n small the parametric approach can be better, thanks to the rich *a priori* knowledge. It is because the class of admissible structures of the linear

dynamics is narrowed at the beginning, and only two parameters are estimated. Appropriate selection of λ_n in semiparametric approach (19), e.g.,

$$\lambda_n = \begin{cases} 1, & \text{for } n < 300 \\ 0, & \text{otherwise,} \end{cases}$$

allows $\hat{k}_j^{(semi)}$ to take advantages of both $\hat{k}_j^{(iv)}$ and $\hat{k}_j^{(corr)}$.

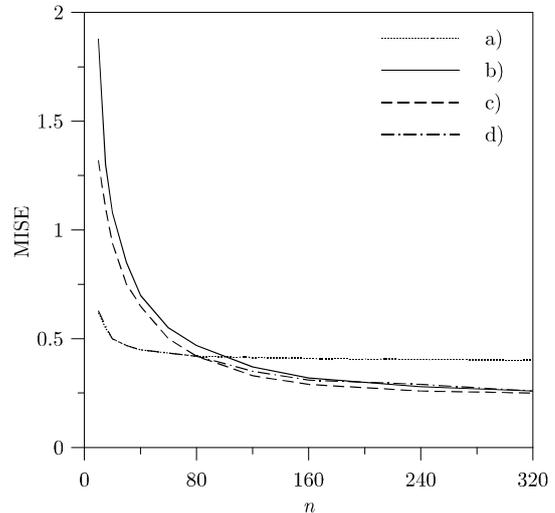


Fig. 8. MISE versus n ; $h_n = 0.8n^{-0.3}$; a) – (9), b) – (6), c) – (10), d) – (11).

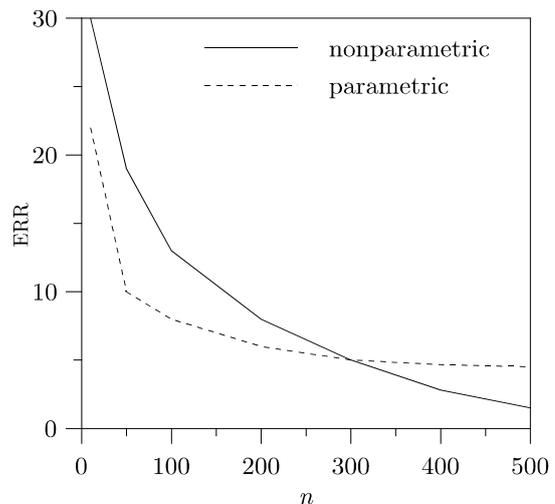


Fig. 9. Estimation error of the linear dynamics.

8. CONCLUSIONS

Simulation examples confirm expectations resulting from theoretical considerations. For small number of observations, semiparametric algorithms behave better than purely nonparametric (6) and (18). Moreover, semiparametric algorithms are much less sensitive to not optimal selecting h_n . In the light of this, the semiparametric methodology is worth further studies.

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